# GHEP kinship exercise 2024: Advanced level 

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## General instructions

This is a multiple-choice test consisting of 20 questions. For each question exactly one alternative is correct. You are free to use whatever software you like, but keep in mind that some programs have built-in conventions (e.g., rounding) that may affect the output. If your answer does not precisely match any of the options, choose the closest one.

## Files needed to complete the test

- cousins-data.txt / cousins-ibd.txt. Data for Part II.
- $\quad$ siblings-data.txt / siblings-ibd.txt. Data for Part III.
- db.txt. Allele frequencies for 23 STR markers. (If your software requires database size, use $N=1000$.)


## Assumptions throughout

- No linkage between markers, no linkage disequilibrium, no deviations from HW equilibrium.
- No drop-outs, drop-ins, silent alleles or mutations.
- Pedigree founders are non-inbred and unrelated to each other.
- The total genetic length of the autosome (chromosomes 1-22) is 3391 cM.


## Some definitions

Homologous alleles are identical by descent (IBD) if they have the same origin within a given pedigree. The IBD coefficients $\left(\kappa_{0}, \kappa_{1}, \kappa_{2}\right)$ of non-inbred individuals $A$ and $B$, are the probabilities of sharing respectively 0,1 and 2 alleles IBD at a random autosomal locus. They are related to the kinship coefficient $\varphi$ by the formula $\varphi=\kappa_{1} / 4+\kappa_{2} / 2$.

The IBD triangle (Figure 1) is a convenient tool for visualising IBD coefficients. Note that, since $\kappa_{0}+\kappa_{1}+\kappa_{2}=1$, any two of them suffice to deduce the third; the choice of $\kappa_{0}$ and $\kappa_{2}$ is simply my personal preference. The online tool QuickPed may be useful for calculating IBD coefficients and plotting them in the IBD triangle.

Traditional coefficients like $\kappa$ and $\varphi$ measure the expected IBD sharing based on the pedigree. In contrast, the realised (or genomic) relatedness between $A$ and $B$ refers to the actual IBD segments they share as a result of recombination (Figure 2). We denote by ( $k_{0}, k_{1}, k_{2}$ ) the actual proportions of the autosome, in terms of genetic length, where they share 0,1 and 2 alleles IBD, respectively. The realised kinship coefficient is given by $\varphi_{R}=k_{1} / 4+k_{2} / 2$.


Figure 1. The IBD triangle. FC=first cousins; G=grandparentgrandchild; H=half sibs; MZ=monozygous twins; $P O=$ parent-
offspring; S=full sibs; U=uncle-nephew; UN=unrelated


Figure 2. An example of the realised IBD sharing between siblings. The chromosome is divided in segments with IBD status 0,1 or 2

## Questions

## Part I: Warm-up

We consider a situation where two individuals, $A$ and $B$, are typed with a tri-allelic marker. The alleles are labelled $1,2,3$, and the allele frequencies are $p_{1}, p_{2}, p_{3}$, respectively.

1. Suppose the marker lies in a region where $A$ and $B$ have IBD status 0 . If $A$ has genotype $1 / 1$, the genotype of $B$ is
a) $2 / 3$
b) $2 / 2$ or $3 / 3$
c) $2 / 2,2 / 3$ or $3 / 3$
d) anything except $1 / 1$
e) anything
2. Suppose the marker lies in a region with $I B D=1$. If $A$ has genotype $1 / 2$, the genotype of $B$ is
a) $1 / 3$ or $2 / 3$
b) $1 / 1,2 / 2,1 / 3$ or $2 / 3$
c) anything except $1 / 2$
d) anything except $3 / 3$
e) anything

In the next three questions, we assume that the genotypes of both individuals are unknown.
3. Given that the marker is in a region with IBD $=2$, the probability that $A$ and $B$ are homozygous for the same allele, is
a) 0
b) $p_{1}^{2}+p_{2}^{2}+p_{3}^{2}$
c) $\quad p_{1}\left(1-p_{1}\right)+p_{2}\left(1-p_{2}\right)+p_{3}\left(1-p_{3}\right)$
d) $p_{1}\left(1-p_{1}\right)^{2}+p_{2}\left(1-p_{2}\right)^{2}+p_{3}\left(1-p_{3}\right)^{2}$
e) 1
4. Given that the marker is in a region with $I B D=1$, the probability of a full match (i.e., $A$ and $B$ have the same genotype) is
a) 0
b) $p_{1}^{2}+p_{2}^{2}+p_{3}^{2}$
c) $\quad p_{1}\left(1-p_{1}\right)+p_{2}\left(1-p_{2}\right)+p_{3}\left(1-p_{3}\right)$
d) $p_{1}\left(1-p_{1}\right)^{2}+p_{2}\left(1-p_{2}\right)^{2}+p_{3}\left(1-p_{3}\right)^{2}$
e) 1
5. Given that the marker is in a region with IBD = 1, the probability of a partial match (i.e., exactly one shared allele) is
a) 0
b) $p_{1}^{2}+p_{2}^{2}+p_{3}^{2}$
c) $\quad p_{1}\left(1-p_{1}\right)+p_{2}\left(1-p_{2}\right)+p_{3}\left(1-p_{3}\right)$
d) $\quad p_{1}\left(1-p_{1}\right)^{2}+p_{2}\left(1-p_{2}\right)^{2}+p_{3}\left(1-p_{3}\right)^{2}$
e) 1

## Part II: A case of cousins

Emma and Carlos are about to get married, but suspect that they are related to each other. They consult a geneticist, who types them with 15 standard STR markers. The resulting genotypes are given in the file cousins-data.txt, along with the physical location of each marker. Allele frequencies can be found in db.txt.
Note: Recall that linkage is to be ignored in LR calculations. The locations are only used in Exercise 8.
6. Use all 15 markers to compute the LR comparing the hypothesis that Emma and Carlos are first cousins, to the unrelated alternative. The total LR, rounded to two decimals, is
a) 0.10
b) 0.75
c) $\quad 1.00$
d) 3.14
e) $\quad 13.14$

In a more exhaustive approach the geneticist compares the following 5 hypotheses, illustrated in Figure 3:

- H1: first cousins
- H2: first cousins once removed
- H3: second cousins
- H4: third cousins
- H5: unrelated


Figure 3. Hypotheses for Question 7
7. The LR comparing the most likely hypothesis with the second most likely, is approximately
a) 1.00
b) $\quad 1.12$
c) $\quad 1.45$
d) 3.61
e) $\quad 10.18$

Not completely satisfied with the STR analysis, Emma and Carlos undergo whole-genome sequencing, and the geneticist uses this to identify regions of IBD sharing between them. The result is given in the file cousins-ibd.txt, reproduced in its entirety below. In all segments Emma and Carlos share one haplotype, i.e., IBD = 1 .

| Chr | startMB | endMB | startCM | endCM |
| ---: | ---: | ---: | ---: | ---: |
| 3 | 55.28 | 98.00 | 71.71 | 108.40 |
| 4 | 187.98 | 189.44 | 199.80 | 202.89 |
| 5 | 170.68 | 180.75 | 174.66 | 197.08 |
| 9 | 101.98 | 112.07 | 101.00 | 112.06 |
| 11 | 37.09 | 98.83 | 55.18 | 99.72 |
| 11 | 122.33 | 131.01 | 127.17 | 143.25 |
| 12 | 8.23 | 15.26 | 20.31 | 32.13 |

8. Of the 15 STR markers, the number that lie in an IBD region is
a) 0
b) 1
c) 2
d) 3
e) 4
9. The observed proportion $k_{1}$ of the autosome with IBD status 1 , is approximately
a) $1.1 \%$
b) $1.8 \%$
c) $2.7 \%$
d) $4.3 \%$
e) $6.3 \%$
10. Of the following relationships, the one whose $\kappa_{1}$ is closest to the observed $k_{1}$ for Emma and Carlos, is
a) first cousins
b) first cousins once removed
c) second cousins
d) third cousins
e) unrelated
11. According to the DNA painter tool, https://dnapainter.com/tools/sharedcmv2, the IBD sharing between Emma and Carlos is not compatible with (i.e., outside the reported range of)
a) first cousins
b) first cousins once removed
c) second cousins
d) third cousins
e) several of the above
12. Figure 4 shows the distribution of IBD segments in 200 simulations of first, second and third cousins. Based on this plot, the observed data is only compatible (in the sense of being inside the $95 \%$ data ellipse) with
a) first cousins (FC)
b) second cousins (SC)
c) third cousins (TC)
d) first and second cousins
e) second and third cousins


Figure 4. IBD distributions for 1st, 2nd and 3rd cousins

## Part III: A case of sibship

This case involves 4 male individuals, labelled $A, B, C$ and $D$. It is believed that all four have the same mother, but the paternities are unclear. Genotypes for A, B, C, D at 23 forensic markers can be found in sibship-data.txt, with allele frequencies in db.txt as before.
13. The LR comparing $A$ and $B$ being full siblings versus half siblings, is approximately
a) 0.68
b) 1.00
c) 431.47
d) 133506.3
e) None of the above
14. The LR comparing $C$ and $D$ being full siblings versus half siblings, is approximately
a) 0.68
b) 1.00
c) 431.47
d) 133506.3
e) None of the above

We now make the following assumptions:
i) all four have the same mother
ii) $\quad C$ and $D$ are full siblings
iii) each pair among $A, B, C, D$ is either half or full siblings, with no further relationships or inbreeding in the pedigree
15. The number of possible hypotheses (pedigrees) connecting $A, B, C, D$ is
a) 5
b) 6
c) 7
d) 8
e) 9
16. Let $H^{*}$ denote the hypothesis that all four are full siblings. Assuming a flat prior on set of hypotheses from the previous question, the posterior probability of $H^{*}$ given the marker data, is approximately
a) 0.00
b) 0.01
c) 0.73
d) 0.99
e) $\quad 1.00$


Figure 5. Segments of identity by descent between alleged siblings $A$ and $B$.

To further investigate the relationship between $A$ and $B$, whole-genome sequencing is performed, and their IBD segments are determined as shown in Figure 5. The coordinates can be found in the file sibs-ibd.txt.
17. In terms of genetic length, the proportion $k_{1}$ of the autosome with IBD status 1 , is
a) 0.25
b) 0.30
c) 0.45
d) 0.50
e) 0.60
18. In the IBD triangle in Figure 6, the realised relationship between $A$ and $B$ corresponds to the point
a) P 1
b) P 2
c) P 3
d) P 4
e) P5


Figure 6. IBD triangle with alternatives for the realised relationship between $A$ and $B$.
19. Based on the IBD segments, the realised kinship coefficient $\varphi_{R}$ between A and B is approximately
a) 0.14
b) 0.19
c) 0.22
d) 0.25
e) 0.26

The histogram in Figure 7 shows the realised kinship coefficient in 5000 simulated pairs of full siblings, closely approximated by a normal distribution with mean $\mu=0.25$ and standard deviation $\sigma=0.018$ (dashed red curve).
20. Compared with the normal approximation for full siblings, the observed $\varphi_{R}$ falls at the
a) Oth percentile
b) 5th percentile
c) 10th percentile
d) 15 th percentile
e) 20th percentile


Figure 7. Simulations of the realised kinship between full siblings

